



## Strapdown Inertial Navigation Technology, Second Edition

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The book provides an up-to-date guide to the techniques and applications of inertial navigation for use by both practicing engineers and post-graduate students. The book satisfies a need for a book on the subject of inertial navigation that provides both an introduction to the techniques involved as well as information on modern technological developments, combined with a more rigorous mathematical treatment for the reader wishing to explore the subject in greater depth.

The text describes the basic concepts of inertial navigation with particular emphasis on modern strapdown system technology, providing detailed information on system mechanizations, instrumentation and computational aspects, design analysis, and applications of such systems. In particular, the text provides up-to-date information on inertial sensor technology and inertial navigation system computational techniques, bringing together the broad experience of the authors within a single volume. The text contains both descriptive passages and also mathematical details where appropriate.

MEMS is the focus of much research and development activity at the present time; this technology offers rugged and reliable sensors with a performance capability that lends itself to integration with satellite navigation systems.

This second edition has been updated in a number of areas to reflect ongoing developments in the field of inertial navigation technology. In addition to a number of refinements covering sensor technology, geodesy, and error modeling, the major additions to the original text are new chapters on MEMS (micro electro-mechanical systems) technology and system applications. A broad range of applications are addressed in a second new chapter, covering ship's inertial navigation, tactical missiles, well bore surveying systems, automobiles, and sightline stabilization systems, to name but a few.

## Preface

### 1 Introduction 1

1.1 Navigation	1
1.2 Inertial navigation	2
1.3 Strapdown technology	3
1.4 Layout of the book	4

<b>2 Fundamental principles and historical developments of inertial navigation</b>	<b>7</b>
2.1 Basic concepts	7
2.2 Summary	11
2.3 Historical developments	11
2.4 The modern-day inertial navigation system	14
2.5 Trends in inertial sensor development	15
<b>3 Basic principles of strapdown inertial navigation systems</b>	<b>17</b>
3.1 Introduction	17
3.2 A simple two-dimensional strapdown navigation system	17
3.3 Reference frames	21
3.4 Three-dimensional strapdown navigation system - general analysis	22
3.4.1 Navigation with respect to a fixed frame	22
3.4.2 Navigation with respect to a rotating frame	24
3.4.3 The choice of reference frame	24
3.4.4 Resolution of accelerometer measurements	24
3.4.5 System example	25
3.5 Strapdown system mechanizations	25
3.5.1 Inertial frame mechanization	26
3.5.2 Earth frame mechanization	28
3.5.3 Local geographic navigation frame mechanization	31
3.5.4 Wander azimuth navigation frame mechanization	34
3.5.5 Summary of strapdown system mechanizations	36
3.6 Strapdown attitude representations	36
3.6.1 Introductory remarks	36
3.6.2 Direction cosine matrix	39
3.6.3 Euler angles	40
3.6.4 Quaternions	42
3.6.5 Relationships between direction cosines, Euler angles and quaternions	45
3.7 Detailed navigation equations	47
3.7.1 Navigation equations expressed in component form	47
3.7.2 The shape of the Earth	49
3.7.3 Datum reference models	51
3.7.4 Variation of gravitational attraction over the Earth	55

<b>4 Gyroscope technology 1</b>	<b>59</b>
4.1 Introduction	59
4.2 Conventional sensors	60
4.2.1 Introduction	60
4.2.2 Fundamental principles	60
4.2.3 Components of a mechanical gyroscope	68
4.2.4 Sensor errors	71
4.2.5 Rate-integrating gyroscope	73
4.2.6 Dynamically tuned gyroscope	77
4.2.7 Flex gyroscope	81
4.3 Rate sensors	84
4.3.1 Dual-axis rate transducer (DART)	84
4.3.2 Magnetohydrodynamic sensor	86
4.4 Vibratory gyroscopes	88
4.4.1 Introduction	88
4.4.2 Vibrating wine glass sensor	89
4.4.3 Hemispherical resonator gyroscope	91
4.4.4 Vibrating disc sensor	93
4.4.5 Tuning fork sensor	94
4.4.6 Quartz rate sensor	94
4.4.7 Silicon sensor	96
4.4.8 Vibrating wire rate sensor	98
4.4.9 General characteristics of vibratory sensors	99
4.5 Cryogenic devices	100
4.5.1 Nuclear magnetic resonance gyroscope	100
4.5.2 SARDIN	103
4.6 Electrostatically suspended gyroscope	103
4.7 Other devices for sensing angular motion	105
4.7.1 Fluidic (flueric) sensors	105
4.7.2 Fluxgate magnetometers	107
4.7.3 The transmission line gyroscope	112
<b>5 Gyroscope technology 2</b>	<b>115</b>
5.1 Optical sensors	115
5.1.1 Introduction	115
5.1.2 Fundamental principles	116
5.1.3 Ring laser gyroscope	118
5.1.4 Three-axis ring laser gyroscope configuration	126
5.1.5 Fibre optic gyroscope	126

5.1.6 Photonic crystal optical fibre gyroscope	137
5.1.7 Fibre optic ring resonator gyroscope	140
5.1.8 Ring resonator gyroscope	142
5.1.9 Integrated optical gyroscope	143
5.2 Cold atom sensors	143
5.2.1 Introduction	143
5.2.2 Rotation sensing	144
5.2.3 Measurement of acceleration	145
5.2.4 Gravity gradiometer	146
5.3 Summary of gyroscope technology	148
<b>6 Accelerometer and multi-sensor technology</b>	<b>153</b>
6.1 Introduction	153
6.2 The measurement of translational motion	153
6.3 Mechanical sensors	155
6.3.1 Introduction	155
6.3.2 Principles of operation	155
6.3.3 Sensor errors	156
6.3.4 Force-feedback pendulous accelerometer	157
6.3.5 Pendulous accelerometer hinge elements	159
6.3.6 Two-axes force-feedback accelerometer	160
6.3.7 Open-loop accelerometers	161
6.4 Solid-state accelerometers	161
6.4.1 Vibratory devices	162
6.4.2 Surface acoustic wave accelerometer	163
6.4.3 Silicon sensors	165
6.4.4 Fibre optic accelerometer	168
6.4.5 Optical accelerometers	173
6.4.6 Other acceleration sensors	173
6.5 Multi-functional sensors	174
6.5.1 Introduction	174
6.5.2 Rotating devices	174
6.5.3 Vibratory multi-sensor	178
6.5.4 Mass unbalanced gyroscope	179
6.6 Angular accelerometers	182
6.6.1 Liquid rotor angular accelerometer	183
6.6.2 Gas rotor angular accelerometer	184
6.7 Inclinometers	185
6.8 Summary of accelerometer and multi-sensor technology	186

<b>7 MEMS inertial sensors</b>	<b>189</b>
7.1 Introduction	189
7.2 Silicon processing	192
7.3 MEMS gyroscope technology	193
7.3.1 Introduction	193
7.3.2 Tuning fork MEMS gyroscopes	195
7.3.3 Resonant ring MEMS gyroscopes	202
7.4 MEMS accelerometer technology	205
7.4.1 Introduction	205
7.4.2 Pendulous mass MEMS accelerometers	206
7.4.3 Resonant MEMS accelerometers	207
7.4.4 Tunnelling MEMS accelerometers	209
7.4.5 Electrostatically levitated MEMS accelerometers	210
7.4.6 Dithered accelerometers	212
7.5 MOEMS	212
7.6 Multi-axis/rotating structures	212
7.7 MEMS based inertial measurement units	213
7.7.1 Silicon IMU	213
7.7.2 Quartz IMU	214
7.8 System integration	215
7.9 Summary	216
<b>8 Testing, calibration and compensation</b>	<b>219</b>
8.1 Introduction	219
8.2 Testing philosophy	220
8.3 Test equipment	221
8.4 Data-logging equipment	222
8.5 Gyroscope testing	223
8.5.1 Stability tests - multi-position tests	223
8.5.2 Rate transfer tests	226
8.5.3 Thermal tests	231
8.5.4 Oscillating rate table tests	233
8.5.5 Magnetic sensitivity tests	233
8.5.6 Centrifuge tests	235
8.5.7 Shock tests	237
8.5.8 Vibration tests	238
8.5.9 Combination tests	241
8.5.10 Ageing and storage tests	24

8.6 Accelerometer testing	242
8.6.1 Multi-position tests	244
8.6.2 Long-term stability	244
8.6.3 Thermal tests	246
8.6.4 Magnetic sensitivity tests	246
8.6.5 Centrifuge tests	247
8.6.6 Shock tests	250
8.6.7 Vibration tests	250
8.6.8 Combination tests	251
8.6.9 Ageing and storage tests	252
8.7 Calibration and error compensation	253
8.7.1 Introduction	253
8.7.2 Gyroscope error compensation	254
8.7.3 Accelerometer error compensation	254
8.7.4 Further comments on error compensation	255
8.8 Testing of inertial navigation systems	255
8.9 Hardware in the loop tests	259
<b>9 Strapdown system technology 263</b>	
9.1 Introduction	263
9.2 The components of a strapdown navigation system	263
9.3 The instrument cluster	264
9.3.1 Orthogonal sensor configurations	264
9.3.2 Skewed sensor configurations	265
9.3.3 A skewed sensor configuration using dual-axis gyroscopes	266
9.3.4 Redundant sensor configurations	268
9.4 Instrument electronics	269
9.5 The attitude computer	271
9.6 The navigation computer	272
9.7 Power conditioning	274
9.8 Anti-vibration mounts	274
9.9 Concluding remarks	274
<b>10 Inertial navigation system alignment 277</b>	
10.1 Introduction	277
10.2 Basic principles	278
10.2.1 Alignment on a fixed platform	278
10.2.2 Alignment on a moving platform	280
10.3 Alignment on the ground	282

10.3.1	Introduction	282	
10.3.2	Ground alignment methods	283	
10.3.3	Northfinding techniques	287	
10.4	In-flight alignment	289	
10.4.1	Introduction	289	
10.4.2	Sources of error	289	
10.4.3	In-flight alignment methods	289	
10.5	Alignment at sea	300	
10.5.1	Introduction	300	
10.5.2	Sources of error	300	
10.5.3	Shipboard alignment methods	301	
<b>11</b>	<b>Strapdown navigation system computation</b>	<b>309</b>	
11.1	Introduction	309	
11.2	Attitude computation	310	
11.2.1	Direction cosine algorithms	311	
11.2.2	Rotation angle computation	315	
11.2.3	Rotation vector compensation	316	
11.2.4	Body and navigation frame rotations	318	
11.2.5	Quaternion algorithms	319	
11.2.6	Orthogonalisation and normalisation algorithms	322	
11.2.7	The choice of attitude representation	324	
11.3	Acceleration vector transformation algorithm	324	
11.3.1	Acceleration vector transformation using direction cosines	325	
11.3.2	Rotation correction	326	
11.3.3	Dynamic correction	328	
11.3.4	Acceleration vector transformation using quaternions	329	
11.4	Navigation algorithm	329	
11.5	Summary	332	
<b>12</b>	<b>Generalized system performance analysis</b>	<b>335</b>	
12.1	Introduction	335	
12.2	Propagation of errors in a two-dimensional strapdown navigation system	336	
12.2.1	Navigation in a non-rotating reference frame	336	336
12.2.2	Navigation in a rotating reference frame	337	
12.2.3	The Schuler pendulum	339	

12.2.4 Propagation of errors in a Schuler tuned system	340
12.2.5 Discussion of results	341
12.3 General error equations	342
12.3.1 Derivation of error equations	342
12.3.2 Discussion	346
12.4 Analytical assessment	350
12.4.1 Single channel error model	350
12.4.2 Derivation of single channel error propagation equations	352
12.4.3 Single-channel error propagation examples	358
12.5 Assessment by simulation	360
12.5.1 Introductory remarks	360
12.5.2 Error modeling	361
12.5.3 Simulation techniques	363
12.6 Motion dependence of strapdown system performance	365
12.6.1 Manoeuvre-dependent error terms	366
12.6.2 Vibration dependent error terms	368
12.7 Summary	374
<b>13 Integrated navigation systems</b>	<b>377</b>
13.1 Introduction	377
13.2 Basic principles	378
13.3 External navigation aids	379
13.3.1 Radio navigation aids	379
13.3.2 Satellite navigation aids	384
13.3.3 Star trackers	391
13.3.4 Surface radar trackers	393
13.4 On-board measurements	394
13.4.1 Doppler radar	394
13.4.2 Magnetic measurements	395
13.4.3 Altimeters	396
13.4.4 Terrain referenced navigation	397
13.4.5 Scene matching	398
13.4.6 Continuous visual navigation	399
13.5 System integration	401



13.6 Application of Kalman filtering to aided inertial navigation systems	402
13.6.1 Introduction	402
13.6.2 Design example of aiding	403
13.7 INS-GPS integration	409
13.7.1 Uncoupled systems	411
13.7.2 Loosely coupled integration	412
13.7.3 Tightly coupled integration	413
13.7.4 Deep integration	415
13.7.5 Concluding remarks	416
13.7.6 INS aiding of GPS signal tracking	416
13.8 Multi-sensor integrated navigation	417
13.9 Summary	418
<b>14 Design example 421</b>	
14.1 Introduction	421
14.2 Background to the requirement	422
14.3 The navigation system requirement	423
14.3.1 Navigation data required	423
14.3.2 Operating and storage environment	423
14.3.3 Performance	424
14.3.4 System reaction time	425
14.3.5 Physical characteristics	425
14.4 Why choose strapdown inertial navigation?	426
14.5 Navigation system design and analysis process	426
14.5.1 Introduction	426
14.5.2 Choice of system mechanization	427
14.5.3 Error budget calculations	428
14.5.4 System alignment	433
14.5.5 Choice of inertial instruments	434
14.5.6 Computational requirements	436
14.5.7 Electrical and mechanical interfaces	437
14.6 Testing, calibration and compensation requirements	438
14.7 Performance enhancement by aiding	438
14.8 Concluding remarks	439
<b>15 Alternative applications of IN sensors and systems 441</b>	
15.1 Introduction	441

15.2 Borehole surveying	442
15.2.1 Introduction	442
15.2.2 Historical background	443
15.2.3 Inertial survey system	445
15.2.4 System design requirements	446
15.2.5 System design issues	447
15.2.6 System calibration and test	451
15.2.7 Concluding remarks	452
15.3 Ship's inertial navigation systems (SINS)	453
15.3.1 NATO SINS	454
15.4 Vehicle stabilization and control	456
15.4.1 Autopilots	456
15.4.2 Passive missile roll control (rollerons)	462
15.4.3 Intelligent transport systems - automotive applications	464
15.4.4 Intelligent transport systems - trains	467
15.4.5 Personal transport	467
15.5 Equipment stabilization	469
15.5.1 Aero-flexure compensation	470
15.5.2 Laser beam director	475
15.5.3 Laser radar	479
15.5.4 Seeker-head stabilization	482
15.5.5 Sightline stabilization	487
15.5.6 Relative angular alignment	491
15.5.7 Calibration and measurement	493
15.6 Geodetic and geophysical measurements and observation of fundamental physical phenomena	495
15.7 Other applications	499
15.7.1 Moving-map displays	499
15.7.2 Safety and arming units	502
15.7.3 Aircraft ejection seats	503
15.7.4 Agricultural survey	505
15.7.5 Artillery pointing	505
15.7.6 Other unusual applications	507
15.8 Concluding remarks	508
<b>Appendix A Kalman filtering</b>	<b>511</b>
<b>Appendix B Inertial navigation system error budgets</b>	<b>519</b>

<b>Appendix C Inertial system configurations</b>	<b>523</b>
<b>Appendix D Comparison of GPS and GLONASS satellite navigation systems</b>	<b>529</b>
<b>List of symbols</b>	<b>535</b>
<b>Glossary of principal terms</b>	<b>539</b>
<b>Index</b>	<b>549</b>

