LOTKA, ALFRED JAMES (1880-1949), by Joel E. Cohen

in:

THE NEW PALGRAVE A DICTIONARY OF ECONOMICS

EDITED BY

JOHN EATWELL

MURRAY MILGATE

PETER NEWMAN

London: Macmillan; New York: Stockton Press, 1987

Volume 3: 245-247

K to P

Lotka, Alfred James (1880–1949). Alfred James Lotka was a many-sided scientist who pioneered the mathematical theory of population. He created the demographic theory of stable populations as a special case of a general theory of renewal. He developed and applied to contemporary demographic data the important concepts of net rate of reproduction and intrinsic rate of natural increase. He created and analysed mathematical models of predation and competition that remain influential in theoretical ecology and proposed a comprehensive physico-chemical view of evolution. He also evaluated the expected value of lifetime earnings of workers of specified ages, analysed mathematical models for the epidemiology of malaria, and calculated rank-size distributions of scientific productivity.

Lotka was born on 2 March 1880 to French-speaking parents of American citizenship in Lemberg, Austria (now Lwów, Ukrainian SSR). He received his early education in France, Germany and England. From Mason College, Birmingham University, he received the BSc in 1901 and the DSc in 1912. During a year's study of chemistry at Leipzig University in 1901–2, he developed concepts for a mathematical theory of evolution.

He came to the United States in 1902 and worked as an industrial chemist. In 1908, he registered at Cornell University as a doctoral candidate in physics and mathematics, but left in 1909 with the degree of AM. After working as an examiner at the US Patent Office, as a physicist at the US Bureau of Standards, as a freelance writer, as an editor of the *Scientific American* Supplement, and as a chemist at the General Chemical Company, he accepted in 1922 a temporary research appointment in Raymond Pearl's Human Biology group at Johns Hopkins University. Between 1922 and 1924, he completed his magnum opus, *Elements of Physical Biology* (1925a), putting flesh on the bones of ideas he had developed over the preceding quarter century.

From 1924 until his retirement in 1948, Lotka worked for the Metropolitan Life Insurance Company in New York City as supervisor of mathematical research in the Statistical Bureau (1924–33), as general supervisor (1933–4), and as assistant statistician (1934–48). He married Romola Beattie on 5 January 1935; they had no children. He was president of the Population Association of America in 1938–9, president of the American Statistical Association in 1943, and vice-president of the International Union for the Scientific Investigation of Population Problems. In retirement, Lotka revised and translated portions of his *Théorie analytique des associations biologiques* (1934, 1939c). After an illness of a few weeks, he died in Red Bank, New Jersey, on 5 December 1949.

Lotka's more than one hundred scientific papers and five books range widely, from the mathematics, physics and chemistry of his early training, to fields some of which he helped to create, including theoretical and applied demography, ecology, epidemiology, other mathematical social sciences including economics, and operations research. He was a gifted and engaging expositor.

In 1907, Lotka analysed homogeneous density-independent populations closed to migration and growing at a given rate, in which individuals are subject to a given schedule of mortality. Lotka supposed that the age structure, that is, the proportions of individuals in each age group, is independent of time, and expressed the age structure in terms of the given growth rate and mortality schedule. Estimating the growth rate and the mortality schedule from reports of the Registrar General of England and Wales for 1871–80, Lotka showed that the predicted per capita rates of birth and death and predicted age structure agree well with the corresponding observations.

Lotka entitled his major paper of 1907 containing this analysis 'Studies on the mode of growth of material aggregates'. He followed the analysis immediately by a model of isothermal monomolecular reactions. To explain this juxtaposition, he 'recognized the problem of chemical dynamics as a special case of a wider problem: ... [namely,] the study of the laws governing the distribution of matter among complexes of any specified kind, as determined by their general physical character'. This wider problem, he wrote, 'may be taken to represent the quantitative formulation of the problem of evolution in its most general terms'. He devoted much of the rest of his scientific career to amplifying this perspective of evolution both in abstract generality and in particular contexts.

In 1911, Francis Robert Sharpe (1870–1948) and Lotka showed that the time-invariant age structure, which Lotka had previously analysed, is stable (*see* STABLE POPULATION THEORY). These articles of 1907 and 1911 form the core of Lotka's contributions to population analysis.

In 1934 and 1939, Lotka published the two parts of *Théorie analytique des associations biologiques, I: Principes; II: Analyse démographique avec application particulière à l'espèce humaine.* The latter (1939c) summarizes his contributions to the population analysis of a single, principally the human, species. The book treats the theory of stable populations, including the solution of the renewal equation, and of logistically growing populations; the progeny of a population element; indices of population growth; fecundity by birth order and family size; orphanhood and family composition; and the extinction of a line of descent. Nearly half a century later, the central ideas of demography, as they are used by most demographers, remain close to those Lotka codified in 1939.

Starting in 1910, Lotka described chemical reactions that display damped oscillations. In 1920, he discovered a pair of nonlinear first order differential equations that display undamped oscillations and interpreted them both chemically and in terms of plant-herbivore interactions. Independently, in 1926, the Italian mathematician Vito Volterra (1860–1940) published a model for one species feeding on another that is mathematically identical to Lotka's 1920 model of undamped oscillations. Others of Volterra's models fell within the framework of Lotka's general theories for the interactions of chemical or biological species.

Virtually all ecologists now refer to the system $dx/dt = x(a - bx - cy)$, $dy/dt = y(f - gx - hy)$, where x and y are the abundances or biomasses of two species competing for a common resource, and a, b, c, f, g, h are positive parameters, as the Lotka–Volterra equations. Volterra analysed these equations in 1926 and Lotka in 1932 by different methods.

Lotka concluded a 1932 analysis of the Lotka–Volterra equations by remarking that 'It is perhaps hardly to be expected that concrete examples of the law of growth for two populations here discussed shall be found in nature', a warning many subsequent ecologists who have used the equations have forgotten. He suggested that an experimental realization of the equations would be 'interesting'. At the same time, 1932, the Russian experimentalist G.F. Gause (b. 1910) published microbiological experiments consistent with the Lotka–Volterra equations. The theory of the stability of the Lotka–Volterra equilibria and his own experiments led Gause to formulate, and to attribute to Lotka and Volterra, a principle of competitive exclusion that now bears Gause's name.

Lotka followed contemporary economic thought (he was a member of the Royal Economic Society) and sought to contribute to it. In the same 1932 paper, Lotka suggested that

> the treatment which has here been developed in the analysis of the growth of multiple populations, may find more immediate application in the field of economics ... Cournot's treatment of the problem of competition has been criticized on the ground that ... any one competitor who should possess the slightest advantage over the others,

would ultimately displace them entirely, and hold the field in absolute monopoly.

Lotka showed how the spatial dispersion of competitors avoided this criticism.

Of all his works, Lotka was proudest of *Elements of Physical Biology* (1925a). When the book was reissued in 1956, Herbert A. Simon saw in its exposition of systems of differential equations, stability of equilibria and comparative statics 'many of the sources of Samuelson's analysis of the relations of statics and dynamics in his *Foundations* – a debt which Samuelson acknowledges' (Simon, 1956). He added, 'It is easy to show that much that has happened in mathematical social science in the thirty years since the publication of the first edition of *Elements of Physical Biology* lies in directions along which the book points.' His final assessment is one that many readers of Lotka share: 'When I weary of reading Lotka for his mathematics, I read him for sheer delight, and for the broad perspective he gives me of the world in which I live.'

JOEL E. COHEN

See also PREDATOR-PREY MODELS; STABILITY; STABLE POPULATION THEORY; VOLTERRA, VITO.

SELECTED WORKS

1907. Studies on the mode of growth of material aggregates. *American Journal of Science* 24, 199–216, 375.

1911. (With F.R. Sharpe.) A problem in age-distribution. *Philosophical Magazine* 21, 435–8.

1925a. *Elements of Physical Biology*. Baltimore: Williams and Wilkins. Reprinted as *Elements of Mathematical Biology*, New York: Dover Publications, 1956; Lotka's own list of his scientific papers is appended to this reprint, but it is incomplete.

1925b. (With L.I. Dublin.) On the true rate of natural increase as exemplified by the population of the United States, 1920. *Journal of the American Statistical Association* 20, 305–39.

1928. The progeny of a population element. *American Journal of Hygiene* 8(6), November, 875–901.

1930. (With L.I. Dublin.) *The Money Value of a Man*. New York: Ronald Press Co. Revised edn, 1946.

1934. *Théorie analytique des associations biologiques. Pt I: Principes.* Actualités Scientifiques et Industrielles No. 187, Paris: Hermann et Cie.

1936. (With L.I. Dublin.) *Length of Life, a Study of the Life Table.* New York: Ronald Press. Revised edn, 1949 (with L.I. Dublin and M. Spiegelman).

1937. (With L.I. Dublin.) *Twenty-Five Years of Health Progress: a Study of the Mortality Experience among the Industrial Policyholders of the Metropolitan Life Insurance Company, 1911 to 1935.* Metropolitan Life Insurance Company. Supplement: Health Progress 1936–45.

1939a. A contribution to the theory of self-renewing aggregates, with special reference to industrial replacement. *Annals of Mathematical Statistics* 10(1), March, 1–25.

1939b. On an integral equation in population analysis. *Annals of Mathematical Statistics* 10(2), June, 144–61.

1939c. *Théorie analytique des associations biologiques. Pt II: Analyse démographique avec application particulière à l'espèce humaine.* Actualités Scientifiques et Industrielles No. 780, Paris: Hermann et Cie.

1948. Application of recurrent series in renewal theory. *Annals of Mathematical Statistics* 19(2), June, 190–206.

BIBLIOGRAPHY

Gause, G.F. 1964. *The Struggle for Existence*. New York: Hafner.

Kingsland, S.E. 1985. *Modeling Nature: Episodes in the History of Population Ecology*. Chicago: University of Chicago Press.

Notestein, F.W. 1982. Demography in the United States: a partial account of the development of the field. *Population and Development Review* 8(4), December, 651–87.

Simon, H.A. 1959. Review of *Elements of Mathematical Biology* by Alfred J. Lotka. *Econometrica* 27(3), July, 493–5.

lotteries. *See* EXPECTED UTILITY HYPOTHESIS.

Get the best of The New Yorker in your in-box every day. Submit. Privacy Policy.Â The New Yorker has made its coronavirus news coverage and analysis free for all readers. A Critic at Large. The Unruly Genius of Joyce Carol Oates.