New proofs for the perimeter and area of a circle

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Abstract: In this brief work, the authors confirmed the existing formulae for the perimeter and area of a circle in a different approach.

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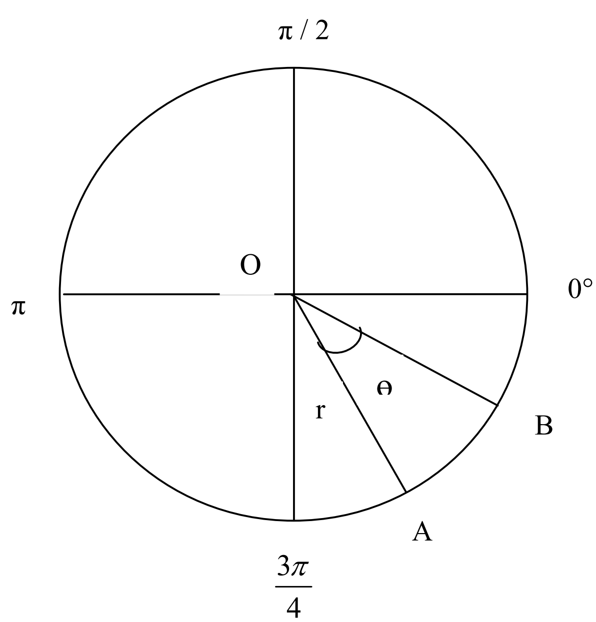
Introduction:

Geometry stands for**:** *geo which means earth and metria which means measure* (Greek). A major contributor to the field of geometry was Euclid – 325 BC who is typically known as the Father of Geometry and is famous for his works called The Elements. As one progresses through the grades, Euclidian geometry (Plane Geometry) is a big part of what is studied. However, non-Euclidean geometry will become a focus in the later grades and college math. Simply put, geometry is the study of the size, shape and position of 2 dimensional shapes and 3 dimensional figures. However, geometry is used daily by almost everyone. In geometry, one explores spatial sense and geometric reasoning. Geometry is found everywhere: in art, architecture, engineering, robotics, land surveys, astronomy, sculptures, space, nature, sports, machines, cars and much more. When taking geometry, spatial reasoning and problem solving skills will be developed. Geometry is linked to many other topics in math, specifically measurement and is used daily by architects, engineers, architects, physicists and land surveyors just to name a few. In the early years of geometry the focus tends to be on shapes and solids then moves to properties and relationships of shapes and solids and as abstract thinking progresses, geometry becomes much more about analysis and reasoning. Geometry is in every part of the curriculum K -12 and through to college and university. Since most educational jurisdictions use a spiraling curriculum, the concepts are re-visited throughout the grades advancing in level of difficulty. Typically in the early years, learners identify shapes and solids, use problem solving skills, deductive reasoning, understand transformations, symmetry and use spatial reasoning. Throughout high school there is a focus on analyzing properties of two and three dimensional shapes, reasoning about geometric relationships and using the coordinate system. Studying geometry provides many foundational skills and helps to build the thinking skills of logic, deductive reasoning, analytical reasoning and problem solving to name just a few. Some of the tools often used in geometry include: Compass, protractors squares, graphing calculators, geometer's sketchpad, rulers etc.

In this article we are going to see a proof that area and perimeter of a circle are not accurate but only approximate.



Consider a circle with radius r and centre at O. In the circle, consider the curve path AB which is very very small in length.

"Any very small portion of a curved path can be approximated as a straight line"

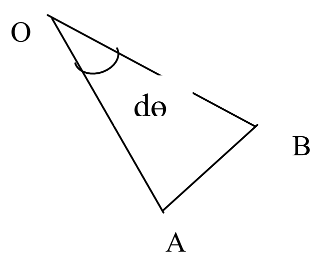
Using the above concept, the curved path AB is approximated as a straight line.

As AB is a very short line, angle AOB is very very small and then OAB is approximated as a right angled triangle, right angle at A and B.

i.e. Angle OAB = Angle OBA = 90^o (approximately) (1)

Let $d\theta$ = Angle AOB = Angle BOA (2)

The right angled triangle is shown below.



In the above figure

Angle OAB = Angle OBA = 90^o (approximately) and $d\theta$ = Angle AOB = Angle BOA

Also $Sind\theta = \frac{AB}{OA} = \frac{AB}{OB}$ (3)

Let AB = dp and OA = OB = r.

Then $Sind\theta = \frac{dp}{r}$ (4)

As $d\theta <<< 1$

$d\theta = 0^o$ (approximately) (5)

As $d\theta = 0^o$ (approximately)

$Sind\theta \approx d\theta$ [1-3] (6)

From (4) and (6), $d\theta = \frac{dp}{r}$ (approximately)

i.e. $dp = rd\theta$ (approximately) (7)

(7) is the equation for the length of the curved path AB.

Integrating equation (7) as θ varies from 0 to 2π, we get the perimeter of the circle

$$p = r\int_{0}^{2\pi} d\theta \quad \text{(approximately)} \qquad (8)$$

i.e. $$p = 2\pi r \quad \text{(approximately)} \qquad (9)$$

Area of the Circle

Consider the approximations

Angle OAB = Angle OBA = 90^{o} (approximately) and Angle AOB = Angle BOA is a very small angle.

Area of the right angled triangle OAB = $\frac{1}{2}$ x base x height

$= \frac{1}{2}$ x AB x OA

$= \frac{1}{2}$ x dp x r (10)

Use (7) in (10), we get

Area of right angled triangle OAB, $dA = \frac{1}{2} r^2 d\theta$ (approximately) (11)

Integrating equation (11) as θ varies from 0 to 2π, we get

Area of the circle, $A = \frac{1}{2} r^2 \int_{0}^{2\pi} d\theta$ (approximately)

i.e. Area of the circle, $A = \pi r^2$ (approximately) (12)

Since the right angles Angle OAB and Angle OBA and the angle at the centre, Angle AOB are approximate, the length of the arc AB i.e. dp and the area of the triangle OAB i.e. dA are also approximate not accurate.

Hence the perimeter $2\pi r$ and the area $\pi \mathrm{r}^2$ of a circle are not accurate but only approximate values.

Discussion:

There are different ways for both perimeter and area of circle. But the author's findings are entirely new and novel. The similar approach may be attempted in other areas of geometry and mathematics. The existing formulae were geometrically deduced. But the author's results were derived geometrically and trigonometrically using calculus.

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