The Book Review Column[1]
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Welcome to the Book Reviews Column. We hope to bring you at least two reviews of books every month. In this column four books are reviewed.

1. **Algorithms and Theory of Computation Handbook** edited by Mikhail Atallah. Reviewed by William Gasarch. This is a new handbook, published in 1999. It is not an update of the MIT handbook that came out in 1990. It is pitched at a lower level.
2. **Handbook of Combinatorics (in two Volumes)** edited by R.L. Graham, M. Grötschel, L. Lovász. Reviewed by William Gasarch. This handbook contains a great deal of material that is useful to theorists. It is written for a mathematical mature audience.
3. **Probabilistic Combinatorics and Its Applications** edited by Béla Bollobás. Reviewed by Danny Krizanc. This book consists of a series of introductory survey articles on topics in probablistic combinatorics and its applications. The emphasis throughout the book is on techniques, with sufficient examples to show their usefulness.
4. **Spectral Graph Theory** by Fan Chung. Reviewed by Jacob Lurie. This book looks at the interplay between a graph and the mathematical properties of its adjacency matrix. It draws motivation from rather advanced mathematics, though knowledge of this mathematics is not strictly neccessary to read it.

I am looking for reviewers for the following books

If you want a FREE copy of one of these books in exchange for a review, then email me at gasarchcs.umd.edu

The following are DIMACS workshop books which are collections of articles.

1. Randomization Methods in Algorithm Design.
2. Microsurveys in Discrete Probability.
3. Mathematical Support of Molecular Biology.
4. Multichannel Optical Networks: Theory and Practice.
5. Networks in Distributed Computing.
6. Advances in Switching Networks.
7. Network Design: Connectivity and Facilities Location.

Review of *Algorithms and Theory of Computation Handbook* edited by
Mikhail Atallah
Published by CRC press in 1999
Number of pages 1296[2]

[2] The ith page of chapter j is numbered j-i, hence the number of pages is not obvious. I got the number of pages off of amazon.com

Hardcover, \$89.00
ISBN number 0-849-32649-4

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1 Overview

One uses a handbook by looking up things that you always wanted to know or that come up. Hence, I decided to review this handbook by asking a random collection of theorists (the editors of SIGACT NEWS and theorists in the Maryland-Wash DC area) to email me questions they are either curious about or think ought to be in a handbook. I comment on how well the book does on answering each one, and then summarize how well the book did overall. For each question asked I looked rather carefully in the table of contents and the index; hence, if I say 'the book did not have anything on topic X' and in reality it does, then the books organization is at fault.

This book is intended for undergraduates who have had the basic undergraduate theory courses, and for the computer professional. There are 48 chapters. For the table of contents the reader is referred to the website http://www.crcpress.com.

Section 2 comments on the questions that the handbook did well on, and Section 3 comments on those that the handbook did not do so well on. Realize that this is a subjective judgement.

2 Questions the handbook did well on

1. What problems are easily parallelizable? There is one chapter on parallel complexity and one on parallel algorithms. The one on parallel complexity covers P vs. NC and P-completeness. The one on parallel algorithms uses for its model an algol-like language. Since these models are incompatible they may give different notions of parallelizable. This is fine since there are different definitions and the issue is not resolved yet.

2. How does one do primitive operations in Computational Geometry such as determining what side of a given line a given point is on? Do they discuss convex hulls? Maxima? Triangulation? Delaunay triangulation? Vornoi diagrams? Closest pair? All of these are discussed. Two dimensional convex hull is solved using the Graham-scan algorithm. For higher dimension a theorem is stated but not proved. Maxima is done in two dimensions and then in all dimensions. An $O(n \log n)$ triangulation algorithm is given. The $O(n \log^* n)$ Las Vegas algorithm, and the $O(n)$ algorithm are mentioned but (wisely) not presented.

3. How do you solve recurrences? How about really hard ones? In the chapter ALGORITHM DESIGN AND ANALYSIS TECHNIQUES they deal with recurrences of the form $T(n) = \sum_{i=0}^{k} a_i T(n-i)$ and $T(n) = g(n) + aT(n/b)$. The treatments are fairly short and leave out many cases, but include those relevant for algorithm analysis. In AVERAGE CASE ANALYSIS OF ALGORITHMS they deal with far more sophisticated techniques such as Mellon transforms.

4. What is the best known algorithm for graph planarity? They mention the Hopcroft-Tarjan $O(n)$ algorithm but do not present it.

5. What are Dijkstra's algorithm, Kruskal's algorithm, Prim's algorithm, and Floyd's algorithm? All of these are included. There is a good discussion with algorithms and some analysis. Some advanced results are mentioned: (1) Dijkstra's and Prim's algorithm can be speeded up to $O(|E|+|V|\log|V|)$ with Fibonacci trees, (2) there is a randomized algorithm (due to Karger-Klein-Tarjan) that solves MST in $O(n)$ expected time.

6. What's this I hear about quantum computing? The section on cryptography has four pages on quantum computing and quantum crypto, including some material on Shor's factoring algorithm. Realize that you can't really say much on this topic in four pages.

7. What is the Chernoff bound? The formula and derivation are there.

8. What is the exponent for matrix multiplication? The book discusses the problem and presents the $O(n^{2.81})$ algorithm of Strassen. It mentions but (wisely) does not prove the best known algorithm which is $O(n^{2.376})$ and gives a reference.

9. What is known about primality testing? In the chapter on ENCRYTPTION SCHEMES they give the Solovay-Strassen probablistic algorithm (discovered independently by Lehmer). They also mention the relatively recent result that there are an infinite number of Carmichael numbers. (n is a Carmichael number if n is composite and $(\forall a \neq 0)[a^{n-1} \equiv 1 \pmod{n}]$. These are important since they are counterexamples for a certain conjectured primality algorithm.) They do not mention the probabilistic complexity classes that PRIME is in, nor that it is in NP $\cap$ co-NP.

10. What is known about factoring? In the chapter on CRYPTANALYSIS they outline the Quadratic Sieve algorithm, which works in $\exp(O((\log n)^{\frac{1}{2}}(\log\log n)^{\frac{1}{2}}))$ steps to factor n, and the Number Field Sieve which takes $\exp(O((\log n)^{\frac{1}{3}}(\log\log n)^{\frac{2}{3}}))$.

11. What do they have on scheduling? In particular, do they talk about min tardiness scheduling (minimize the amount by which a job is late)? There is an entire chapter on scheduling. Min tardiness is mentioned in that they discuss Jackson's rule. In addition scheduling comes up briefly in the chapter on on-line algorithms, and in the chapter on integer programming.

12. What are the levels in Chomsky's hierarchy of formal languages? This is given a full treatment.

13. How do the various complexity classes relate to each other? There is a diagram of how P, ZPP, RP, BPP, NP, PP, PH, P^{PP}, and PSPACE relate.

14. What is an LL(1) grammar? LL(2) grammar? They define these terms and give examples and counterexamples.

15. How do you prove Cook's Theorem? How do you prove that HALT is undecidable? Both Cook's Theorem and the undecidability of HALT are proven.

16. Are there machine-independent characterizations of the usual complexity classes? Fagin's theorem, and others like it, are stated but (wisely) not proven.

3 Questions the handbook did not do well on

1. What is known about parallel sorting? The book has parallel quicksort and parallel radix sort but nothing else. No k-round sorting, no expander graphs, no sorting networks, and no AKS.

2. What is NP? What is PSPACE? What are NP's natural complete problems? What is UP? What are UP's natural complete problems? What is the connection between UP and one-way functions? The book talks about P, NP, and PSPACE (including Savitch's theorem). The book does not discuss UP or FewP. Counting classes are discussed. One-way functions are not mentioned (except briefly when talking about hashing). The notions of UP and one-way functions, and the connection, could have been established.

3. What are the known upper and lower bounds for sorting, selection, element distinctness, hi-lo, and similar problems using comparisons? How about using other types of queries?

   They have all the standard sorting algorithms. They do not have the one that is best for number-of-comparisons (See Ford-Johnson's algorithm either in Knuth Volume 3 or American Math Montly Volume 66, 387–389 and also see Manacher's article in JACM 1979, Volume 26). This may be because these algorithms are bad in practice (for example they both use a quadratic number of moves). They have the randomized selection algorithm but do not have the deterministic $O(n)$ algorithm. They do not have element distinctness or hi-lo. They do not consider other types of queries. In short, they have the practical algorithms you might need but neglect questions that are of theoretical interest.

4. Do they mention the following important theorems: (1) DSPACE(n) closed under complementation. (2) If SAT has non-uniform poly-sized circuits then the polynomial hierarchy collapses to Σ_2^p. (3) If SAT $\le_m^p S$ where S is sparse then P = NP. (4) IP = PSPACE. (5) NP is contained in PCP($\log n, 1$). (6) Parity cannot be recognized by poly-size constant depth circuits. (7) There are sets A and B such that $\mathrm{P}^A = \mathrm{NP}^A$ and $\mathrm{P}^B \neq \mathrm{NP}^B$. (8) PERM is #P-complete.

   All of these results are stated but not proven. The proof of (3) is easy enough to include (especially using the modern proof that is a subcase of Ogiwara's–Watnabe theorem on btt-reductions). The proofs of (1), (2), and (7) should have been included.

5. What is a Hamiltonian graph? Eulerian graph? Planar graph? Genus-g graph? They mention that recognizing Hamiltonian graphs is NP-complete; however, they do not say anything about Eulerian graphs at all except their definition. Planar graphs are discussed, however genus-g graphs are not.

6. Do they state or prove the equivalence of DFA's, NFA's, Reg. expressions, Reg grammars? CFG's and PDA's? The chapter on BASIC NOTIONS IN COMPUTATIONAL COMPLEXITY does all the equivalences for regular language. This same chapter defines PDA's. The chapter on FORMAL GRAMMARS AND LANGUAGES defines CFG's, however the connection is not stated.

7. What is a Horn clause? What is Predicate Calculus? What is Propositional Calculus? What is resolution theorem proving? None of this is discussed except for the proof that SAT is NP-complete.

8. What is a garbage collector, what is mark-sweep, why is malloc/free slow? There is nothing on garbage collection.

9. Is there a general theorem on emergent behavior of distributed systems? There is a section on distributed computing, but no such theorem is in the handbook. Nor is this issue discussed.

10. What are the paradigms of Dynamic programming, Divide and Conquer, and Greedy? There are many references to these paradigms in the index, however they all lead to a particular algorithm using these paradigms. There is no discussion of what either one is in general.

11. What is the polynomial hierarchy? What is the arithmetic hierarchy? How do concepts in complexity theory and recursion theory relate? The polynomial hierarchy and many other classes are defined. The arithmetic hierarchy is not defined. There are chapters on computability and on complexity but there is no real strong sense that the two are related.

12. What is Godel's completeness theorem? incompleteness theorem? This is not mentioned.

13. What is an ω-automata? This is not mentioned.

14. What is known about dynamic algorithms for transitive closure? Both semi-dynamic and fully dynamic? There is a chapter on DYNAMIC GRAPH ALGORITHMS but this particular question is not addressed.

15. What is Lenstra's fixed dimensional integer programming method? While there is a good chapter on integer programming, this is not included.

16. Which interesting problems fall into which approximability classes? The chapter on complexity classes defines MAX SNP and mentions that MAX CUT and MAX CLIQUE are complete for MAX SNP (the proof for MAX CLIQUE is given assuming NP $\subseteq$ PCP($\log n, 1$)). No other problems are given and the class MAX SNP is not mentioned in the chapters on approximation algorithms.

17. What is a zero knowledge proof? This is mentioned in passing in the chapter on Electronic Cash, but is not given any real treatment.

18. What is known about matrix inversion? Finding the determinant? Matrix inversion is not mentioned. This is surprising since matrix inversion is equivalent to matrix multiplication, and is covered in standard texts such as Cormen-Leiserson-Rivest. The determinant is mentioned in the context of randomized algorithms, matrix operations, and complexity theory. They do mention that PERM is complete for GapP, while DET is complete for GapL.

4 Summary and Opinion

Questions on basic algorithms tended to have good answers and good commentary on practical concerns. Questions on basic notions in complexity theory tended to have good answers and some more advanced theorems were stated without proof (usually wisely). Theoretical questions that do not have a practical angle were usually not answered. There is nothing on semantics or logic; however, since the book's title is *Algorithms and Theory of Computation Handbook* it is not claiming to cover those areas.

Many of the chapters have a practical bias and the very choice of topics shows a bias towards practice (e.g., a chapter on graph-drawing algorithms, and seven chapters on cryptography and its variants, while only six chapters on complexity theory). Given the intended audience, this is might be appropriate.

The book to compare this to is *The Handbook of Theoretical Computer Science*, edited by Jan Van Leeuwen, MIT press, published in 1990, (paperback available in 1994). Volume A is on ALGORITHMS AND COMPLEXITY and Volume B is on FORMAL MODELS AND SEMANTICS. I will refer to the old handbook as MIT and the new one as CRC. Hence the fair question is "I have Volume A of MIT. Do I need CRC?" The following topics are in CRC but not in MIT: dynamic graph algorithms, graph drawing algorithms, on-line algorithms, comp learning theory, convex optimization, AI search algorithms, and simulated annealing techniques. The topics that are in both books get a deeper treatment in MIT but some of the newer results are stated (but not proven) in CRC but not in MIT. Note that the price of CRC is very cheap ($89.00)

The combination of the index and the table of contents made items in CRC easy to find.

5 Acknowledgments

I would like to thank Dan Engel, Carolyn Gasarch, James Glenn, Lane Hemaspaandra, Samir Khuller, Clyde Kruskal, David Kueker, Songrit Maneewongvatana, Dave Mount, Ian Parberry, and Aaron Rosenzweig for questions and comments.

Review of *Handbook of Combinatorics (in two Volumes)* edited by
R.L. Graham, M. Grötschel, L. Lovász
Published by MIT press in 1996
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ISBN number 0-262-07169-X

Review by
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One uses a handbook by looking up things that you always wanted to know or that come up. Hence, I decided to review this handbook by asking a random collection of theorists (the editors of SIGACT NEWS and theorists in the Maryland-Wash DC area) to email me questions they are either curious about or think ought to be in a handbook. I comment on how well the book does on answering each one, and then summarize how well the book did overall. For each question asked I looked rather carefully in the table of contents and the index; hence, if I say 'the book did not have anything on topic X' and in reality it does, then the books organization is at fault.

This book is intended for undergraduates who have had the basic undergraduate theory courses, and for the computer professional. There are 48 chapters. The book claims that it is intended for "working mathematician or computer scientist" For the table of contents, the reader is referred to

http://mitpress.mit.edu/book-table-of-contents.tcl?isbn=026207169X.

Section 1 comments on the questions that the handbook did well on, and Section 2 comments on those that the handbook did not do so well on. Realize that this is a subjective judgement. Section 3 comments on what is here of interest to theoretical computer scientists.

1 Questions the handbook did well on

1. Is there a short or at least shorter proof of the four-color theorem? The handbook reports that the original proof of Appel and Haken (in 1976) used 1936 configurations; however there is a new proof due to Robertson et al. (1994) that uses only 633 configurations. They still have to be examined via computer. The handbook has many other points of interest about the four-color theorem including some of its history and equivalent formulations.

2. Is there an asymptotic formula for the Stirling numbers of the first or second kind? Generating functions for both types of Stirling numbers are given and a reference for finding out more about Stirling numbers of the first kind is given.

3. How do you actually prove Kuratowski's theorem? They state the theorem and refer to Kuratowski's original paper (in French) and a more recent proof by Thomassen (in English). They also prove the following: a graph is outer-planer iff it does not have K_4 or $K_{2,3}$ as a minor.

4. What do the eigenvalues of a graph tell us about the graph? There is a nice section on this in the chapter TOOLS FROM LINEAR ALGEBRA. They prove several theorems including the following: $\frac{1}{\lambda_{\min}} \leq \chi(G)$. In a different chapter they state but do not prove $\frac{1-\lambda_{\max}}{\lambda_{\min}} \leq \chi(G) \leq 1+\lambda_{\max}$. This was in the chapter COLORING, STABLE SETS, AND PERFECT GRAPHS. The result was hard to find since the page it was on is NOT in the index under "eigenvalues" or anything close. (I found it by accident.)

5. Are well-quasi-orders covered? How about the GMT (Graph minor theorem— the set of graphs under minor ordering forms a well quasi order). Are better-quasi-orders covered? Well quasi orders are in the book and the basic theorems are covered, including Kruskal's Theorem. The full GMT is (wisely) not proven. Better quasi orders are not covered, which is okay if you are a theorist using this book as a reference (I've never seen better quasi orders used in Theoretical computer science) but it still seems like it should be there.

6. What does it have on infinite combinatorics? It has the basic theorems of infinite combinatorics (e.g., infinite Ramsey) and a few more.

7. Is there a primitive recursive bound on the Van Der Waerdan numbers? There is, and this is presented well.

8. What are the known Ramsey numbers? Van Der Waerdan numbers? It has a table of the known Ramsey numbers, but not of the VDW numbers. There are very few VDW numbers known so this is not really a bad thing.

9. What are matroids good for? There is over 100 pages on matroids. The introduction says that it is motivated by matrix theory and graph theory, but doesn't really say where its heading. However, skimming those 100 pages you can see some applications.

10. For which p is the graph $G(n,p)$ almost surely connected? an expander? other properties? The handbook stated a theorem which implies that there is such a probability for both cases. For connectivity they state that if $\lim_{n\to\infty} np - \log n = c$ then $\Pr(G(n,p))$ is connected approaches $e^{-e^{-c}}$. For expander graphs they do not have any theorem, however they do point out that for many second order properties the threshold is not known, hence that may be the case here.

11. Is there a cardinality so large that if A is a set of that cardinality and you color unordered pairs from A you get an uncountable homogeneous set? Yes, and $2^{2^{\aleph_0}}$ suffices. The proof is there too.

12. What is the maximum number of facets of any polytope in d dim of n vertices? The handbook has the answer as $\binom{n-\lfloor 0.5(d+1)\rfloor}{n-d} + \binom{n-\lfloor 0.5(d+2)\rfloor}{n-d}$.

13. How do the degree, genus, and chromatic number relate to each other? A surface of genus g is a sphere with g handles. A graph is of genus g if it can be embedded on a surface of genus g. The handbook states Heawood's formula $\chi(G) \le \lfloor \frac{7+\sqrt{48g+1}}{2} \rfloor$ and proves it for $g \ge 1$. The book states that this is optimal. Note that the $g = 0$ case is the 4-color theorem. The handbook states ad proves Brook's theorem: (1) $\chi(G) \le deg(G) + 1$, (2) if $deg(G) = 2$ then $\chi(G) = deg(G) + 1$ iff G has an odd cycle as a connected component, (3) if $deg(G) \ne 2$ then $\chi(G) = deg(G) + 1$ iff G has a complete graph on $deg(G) + 1$ vertices as a connected component.

14. Given a positive integer n, how many ways can n be partitioned as a sum of positive integers? The handbook gives a generating function for this, some asymptotics, some theorems, and some references.

15. What to they have on Szemerdi's theorem that every set of positive upper density has arithmetic progressions of arbitrary length? They state the theorem and give some background information. They (wisely) do not prove it.

16. It has been said that combinatorics and number theory are cousins. How much material is there on number theory in the book? (I realize that its not really the focus of the book.) There is a chapter on combinatorial number theory which includes sieve methods and Van der Waerden's theorem. In the section on asymptotics they prove that the sum of all the primes $\le n$ approaches $\frac{n^2}{2logn}$.

17. Let $d_2(n)$ be the maximum number of times the distance 1 occurs between n points in R^2. What is known about $d_2(n)$? In the chapter on EXTREMAL PROBLEMS IN COMBINATORIAL GEOMETRY they state that $n^{1+\frac{c}{\log\log n}} < d_2(n) < cn^{\frac{4}{3}}$. Proofs of weaker results, and results in higher dimensions, are sketched.

2 Questions the handbook did not do well on

1. What is the number of triangulations of a convex polygon (Catalan numbers)? The Catalan numbers are in the handbook, but the fact that they are also the number of triangulations of a convex polygon is not.

2. Can some of the graph parameters (chromatic number, crossing number, genus, etc.) be defined in a way to allow fractional values? If so, is the (say) 4.5-color theorem easier to prove than the 4-color theorem? Fractional edge-colorings, fractional matching, and fractional node-covers are mentioned. Fractional chromatic number is not mentioned.

3. Can binomial coefficients be defined for negative numbers? Fractions? Irrational Numbers? Complex numbers? How about multinomial coefficients? None of this seems to be in the handbook. It is hard to tell since the index only gives one reference to "binomial coefficient"

and two to "multinomial coefficient" and both of these are in the section on HISTORY OF COMBINATORICS.

4. Are there applications of Ramsey theory to theoretical computer science? My first attempt to find any yielded none (the index was not very helpful). Later, by accident, I found one in the chapter on PROB. METHOD. And then a few in the chapter on Ramsey theory. The ones they listed were ones I had not heard of before, but other standard ones (e.g., Yao's paper SHOULD TABLES BE SORTED, Manber, Moran, and Snir's APPLICATIONS OF RAMSEY THEOREM TO DECISION TREE COMPLEXITY, Snir's ON PARALLEL SEARCHING, Alon and Maass RAMSEY THEORY AND LOWER BOUNDS FOR BRANCHING PROGRAMS) were not listed. Alon's paper on constructive lower bounds for certain types of Ramsey Numbers was listed but is not an application.

5. Given a number n I want to know the minimal N such that if you 2-color K_N you are guaranteed a monochromatic cycle of length n. This leads to the field of Graph Ramsey Theory. There was some on this but not alot.

6. What can the number of k-sets be for a set of n points? This problem is not addressed.

7. What is the number of cells that a set of k planes (or hyperplanes) creates? This problem is brought up tangentially, but no results are stated.

8. How many 8×8 knight's tours are there? Knight tours are mentioned in the HISTORY chapter, but this question is not addressed.

9. Under what conditions does a tournament have a directed Hamiltonian cycle? This is not addressed.

3 What is of interest to us?

The prior sections of this review viewed a handbook as a reference. In this section we look at the book to see what is of interest to us as computer scientists.

1. There is some nice stuff on Graph Minors and embeddings. GREAT— I don't have to read the Robertson Seymour papers (Graph Minors 1, Graph Minors 2,. . ., Graph Minors 24) to get a sense of the subject.

2. The chapter on Random graphs answered most of my questions on that topic.

3. The chapters on COMBINATORIAL OPTIMIZATION, COMBINATORICS IN OPERATIONS RESEARCH, and COMPUTATIONAL COMPLEXITY look like nice overviews of these fields.

4. The chapter on COMBINATORICS IN COMPUTER SCIENCE is nice but badly titled since COMPUTER SCIENCE or even COMPUTER SCIENCE THEORY is quite broad. It would be more accurate to call it COMBINATORICS FOR PROVING LOWER BOUNDS ON CONCRETE MODELS.

5. The chapters on COMBINATORIAL GAMES and HISTORY OF COMBINATORICS look readable and very interesting. The chapters on ASYMPTOTIC ENUMERATION METHODS looks hard but very interesting.

4 Opinion

This book has very striking PROS and CONS.
PROS: The topics in it are fascinating and there is much that is worth learning. The articles are well written and the choice of topics is reasonable.
CONS: The combination of index and table of contenst was not that good for finding answers to questions. (It should be noted that it is very hard to index a handbook.) I found many things by accident that the index was no help on. In addition there are several topics that should be in the book but appear not to be.
NEUTRAL: The book is on a sophisticated mathematical level. The audience is mathematicians and computer scientists who already know some combinatorics.
FINAL THOUGHTS: In a perfect world I would have a year free from all obligations and would read this book cover to cover. Alas, since we are not in a perfect world, is it worth the $330.00 to have it as a reference and to browse sometimes? I would say its worth having your local library buy it for such purpose.

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Review of
Probabilistic Combinatorics and Its Applications[3]
Editor: Béla Bollobás
Publisher: American Mathematical Society, 1991
ISBN 0-8218-5500-X
Hardcover
Price: $43.00 (US)

Reviewed by: Danny Krizanc
Carleton University, Ottawa, Canada.

1 Overview

Probablistic arguments have been applied in many areas of combinatorics and theoretical computer science ever since Erdös first used one to prove bounds on Ramsey numbers. Applications range from constructing graphs with properties useful in building communication networks to almost uniform generation of random structures for the purpose of approximately solving intractable counting problems. This book consists of a series of introductory survey articles on topics in probablistic combinatorics and its applications. (The articles are derived from lectures given in one of a series of short courses sponsored by the American Mathematical Society.) The topics covered include random and random-like graphs, discrete isoperimetric inequalities, rapidly mixing Markov chains, and finite Fourier methods. The emphasis throughout the book is on techniques, with sufficient examples to show their usefulness.

2 Summary of Contents

There are total of seven articles on various aspects of probabilistic combinatorics. The first and fourth articles are on random graphs and are both written by Béla Bollobás. The first of these articles introduces the two standard models of random graphs: $\mathcal{G}(n, M)$ - uniform over n vertex graphs with M edges and $\mathcal{G}(n, p)$ - n vertex graphs with edges included independently with probability p. (A variation of $\mathcal{G}(n, M)$ where edges are chosen with replacement is also mentioned.) The relationship between the models, the concept of a threshold function for a graph property, and the basic inequalities used to establish thresholds are discussed. Classical results concerning Ramsey numbers, the chromatic number of sparse graphs and the clique number of random graphs are shown. In the second Bollobás article more advanced techniques such as martingale inequalities, Janson's inequality and the Stein-Chen method are introduced and used to show more detailed results concerning the chromatic and clique numbers of random graphs.

The second article by Fan Chung introduces a number of useful properties of graphs and shows relationships between them. These properties include the Ramsey, discrepancy, expansion, eigenvalue and different extremal properties. In many cases, the existence of graphs having these properties was first shown using probabilistic arguments. Much of the chapter is devoted to describing deterministic constructions of such graphs including Paley, coset, Margulis, and Ramanunjan graphs. Applications to communication networks and open problems are discussed.

There is natural correspondence between the space of random graphs and the hypercube whereby graph properties are identified with subsets of the hypercube. Using this correspondence, thresholds for graph properties can be established using the t-boundary $A_{(t)}$ of a subset A of the hypercube, i.e., the set of points within distance t of A. The third article by Imre Leader concentrates on discrete isoperimetric inequalities or bounds on the size of the t-boundary of a set of a given size. Martingale techniques are used to show sharper bounds then those obtained using traditional expectation and variance methods. An application of these inequalities to the chromatic number of random graphs is discussed by Bollobás in his second article.

Problems complete for the class $\#P$ of counting problems (see [5]), such as computing the permanent, network reliability, the volume of a convex body and the partition function in the Ising model, are believed to be intractable and therefore research has concentrated on finding approximate solutions to them. Jerrum, Valiant and Vazirani [4] proved for sets defined by self-reducible relations (e.g., the examples mentioned above), finding a fully polynomial randomized approximate counting scheme is equivalent to almost uniform generation of a random element of the set. The fifth article by Umesh Vazirani focuses on the technique of rapidly mixing Markov chains (i.e., Markov chains that converge rapidly to a stationary distribution which is uniform over the given set) for performing almost uniform generation. Methods for defining a suitable Markov chain and proving its rapid convergence are surveyed. The concept of the conductance of a chain, bounds on the mixing time it implies, and relationships to eigenvalue separation and expansion are discussed. The topic of the sixth article by Martin Dyer and Alan Frieze is the problem of computing the volume of a convex body in $\mathcal{R}^n$. For this problem it can be shown that no deterministic approximation algorithm exists but by exploiting a powerful isoperimetric inequality and the method of rapidly mixing Markov chains an efficient fully polynomial randomized approximation scheme may be derived. The problem is especially important because of the large number of applications it has. Those discussed in the article include integration of non-negative functions over a convex body, counting linear extensions of a partial order, approximate solution to stochastic programming problems and learning a halfspace.

In the final article, Persi Diaconis shows how finite Fourier methods can be applied to bounding the rate of convergence and estimating covering and first hitting times of random walks on graphs with certain symmetry properties (describable using group theory). The most important example of such a graph is the hypercube which has been studied in detail. This paper begins with the basis of Fourier analysis relevant to the study of random walks and then proceeds to several more sophisticated examples.

3 Opinion

This book is recommended for two audiences: experts in the area and researchers not knowledgeable in the area but who are interested in working in the area or who suspect some of the techniques may be applicable to problems they are working on. For the expert the book makes for a good reference, covering the state of the art of the field up to 1991. Each article gives a large number of references to the historical development of each of the areas covered and to the major applications of the methods described. For the novice the book is an excellent primer, introducing the major techniques of probabilistic combinatorics and giving some examples of how these techniques may be applied to a variety of problems. Some of the material is covered in more depth elsewhere (e.g., Bollobás' own book on random graphs [2], Chung's book on spectral graph theory [3] and the book of Alon and Spencer on the probabilistic method [1]) but for most of the topics these surveys are the only ones I know of. The writing is clear and concise throughout.

References

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Review of *Spectral Graph Theory* by
Fan R.K. Chung
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1 Overview

Specifying a graph is equivalent to specifying its adjacency relation, which may be encoded in the form of a matrix. This suggests that study of the adjacency matrix from a linear-algebraic point of view might yield valuable information about graphs. In particular, any invariant associated to the matrix is also an invariant associated to the graph, and might have combinatorial meaning. Spectral graph theory is the study of the relationship between a graph and the eigenvalues of matrices (such as the adjacency matrix) naturally associated to that graph. This book looks at the subject from a geometric point of view, exploiting an analogy between a graph and a Riemannian manifold: Chung defines the *Laplacian* of a graph, a matrix closely related to the adjacency matrix, in analogy with the continuous case and studies the eigenvalues of this Laplacian.

There are several reasons that these eigenvalues may be of interest. On the purely mathematical level, the eigenvalues have the advantage of being an extremely natural invariant which behaves nicely under operations such as Cartesian product and disjoint union. From a combinatorial point of view, the eigenvalues of a graph are related to many other more "discrete" invariants. From a geometric point of view, there are many respects in which the eigenvalues of a graph behave like the spectrum of a compact Riemannian manfiold. For the computationally-minded, the eigenvalues of a graph are easy to compute, and their relationship to other invariants can often yields good approximations to less tractible computations.

2 Summary

The first chapter is devoted to definitions, examples, and basic facts about the spectrum of the graph, and their application to the study of random walks on a graph. It gives a good taste of what the rest of the book is like in terms of the structure of the arguments, and the application is very natural. It is well worth reading for any who wish an introduction to the subject matter.

The next four chapters investigate the relationship between the spectrum of a graph and other properties of a graph which are of greater combinatorial interest. Stress is put on computational applications, and the mathematics is no longer as pretty.

The sixth chapter has a very different flavor: after indicating how the ideas of spectral graph theory may be applied to aid in the construction (or, more properly, the validation) of expander graphs, Chung launches into a discussion of expanders and their applications to computer science. She also gives many examples of constructions of extremal graphs, whose properties are asserted without proof. This chapter is less mathematically involved than the rest of the book, focusing on bringing ideas together rather than expounding on any one in depth; it is a survey of connections that graph theory has with other subjects.

The seventh chapter is again very different from the remainder of the book. It contains a general discussion of symmetric graphs, with emphasis on the properties of their spectrum. In the last section of this chapter, Chung briefly sketches some connections with group representation theory. The mathematics is simple and elegant, making the chapter a pleasure to read.

The last five chapters delve more deeply into the geometric analogy which informs the entire book: Chung investigates the discrete analogues of diffeo-geometric notions like boundary conditions and the heat equation. Though she never completely loses sight of applications, these fade into the background. As a result, this part of the book seems poorly motivated. It should primarily appeal to those who are already familiar with the geometry in a different setting and want to see the "discrete analogue".

3 Mathematics

For the most part, this book requires few prerequisites: elementary graph theory, linear algebra, and group theory. It is a book about graph theory which employs ideas, but not results, from differential geometry. However, since the origin of these ideas is usually not made clear, a background in geometry is extremely helpful for motivational purposes.

Some sections of the book employ more advanced mathematical ideas. Some sections make the analogy with differential geometry explicit by proving both a combinatorial theorem and its geometric analogue: reading these sections requires a reasonable understanding of Riemannian geometry, but only for understanding the geometric results (which are not needed in other sections of the book). Only once are geometric ideas actually used to obtain combinatorial results (section 10.4). Some relationships to still other areas of mathematics are briefly sketched, but no results or ideas from these other fields are required in order to follow the discussion. This can be disappointing at times, since Chung tends to bring one just close enough to an interesting idea to know that it is out there, but not close enough to grasp it. Luckily, she provides ample references for all of these topics.

The proof techniques employed are fairly homogeneous throughout the book: clever manipulation of inequalities and variational arguments (always for polynomials on a finite dimensional vector space). Theorems typically assert an inequality betwen some combinatorial invariants of a graph (such as the degrees of the vertices, girth, or diameter) and a quantity involving one or more of the eigenvalues of some linear transformation associated with the graph. For example, in the last section of the sixth chapter Chung proves that $\omega(G) \leq \sigma(G) \leq \chi(G)$, where $\omega(G)$ is the clique number of G, $\chi(G)$ the chromatic number of G, and $\sigma(G)$ is the extremal value of an eigenvalue-related problem.

4 Opinion

The book promises and delivers a good mix of mathematical ideas. Though devoted for the most part to discrete mathematics, it touches briefly upon many areas of mathematics, including representation theory, algebraic geometry (in the construction of some extremal graphs), and, of course, differential geometry. Unfortunately, some of the connections between these different areas are sketched too briefly to give a reader a good idea of what these connections are. In perhaps the most extreme example of this, section 8.6 is entitled "Determinants and invariant field theory." The latter topic is mentioned in the first line of the section, its connection to graph theory implied but not explained, and is promptly forgotten.

The biggest shortcoming of Chung's book is poor motivation. The intended applications to computer science are clear throughout the first few chapters, but as the book progresses it is easy to lose sight of her objectives. This is particularly frustrating because most theorems express inequalities between various quantities associated to a graph. A nonexpert such as the reviewer is not likely to have an intuition for the relative sizes of these quantities, so the import of the theorem is lost. A similar criticism may be applied to some of the hypotheses. In section 9.2, Chung defines *convex subgraphs* and *strongly convex subgraphs*. No connection between these notions is given, and in the former case the relationship of the definition with convexity in the usual sense is not made clear.

The book contains many typos, but no mathematical errors (to the reviewer's knowledge). Many of the theorems and their proofs have analogues in differential geometry; Chung mentions this several times but generally fails to explain the origin of many of the geometric ideas she applies. As a result, the mathematics is clear but often seems poorly motivated.

This book is appropriate for people who are interested in graph theory and its connections to other branches of mathematics, know a little differential geometry, and are willing to look up some reference to get a fuller picture.

I hope this book can answer that question, as well as what we can do about it, because surely thereâ€™s a better way than what weâ€™ve done in the past. For all that is wrong in education, there are still some positives.Â Did the author (Jim Trelease) even read any of the books he recommended? A parental review (by me) of 22 of the recommendations of â€œPredictable Booksâ€ revealed that a number of the reviewed books (toddler level) were inappropriate. For example, â€œThe Bear with the Swordâ€ and â€œOnce a Mouseâ€ contain violent language inappropriate for children.